Exercises 13, 20.05.2025

1. As discussed in Lecture 8 piezoelectric materials are mechanically softer under the short circuit conditions than under the open circuit conditions. In practice, very often the situation is not purely short circuit or open circuit, but rather the system can be equivalently represented as a capacitor $C = K_{33}\varepsilon_0 S/L$ with the plates connected via resistance R as shown in **Fig.1**. In this case, if a load is applied very abruptly, the charges have no time to pass through the resistor to compensate the electric field in the sample, and the sample behaves as open circuit. At the same time, for a very slow loading, the charges have enough time to pass through the resistor so that the electric field is compensates in the sample and it behaves as short-circuited.

This phenomenon can be investigated in the experiment where a pressure periodic in time $p = p_0 \cos \omega t$ is applied and the variation of the sample thickness is measured $\Delta L = \Delta L_0 \cos(\omega t + \varphi)$. Now the Young modulus is evaluated as the ratio of amplitudes:

$$Y(\omega) = \frac{L p_0}{\Delta L_0}.$$

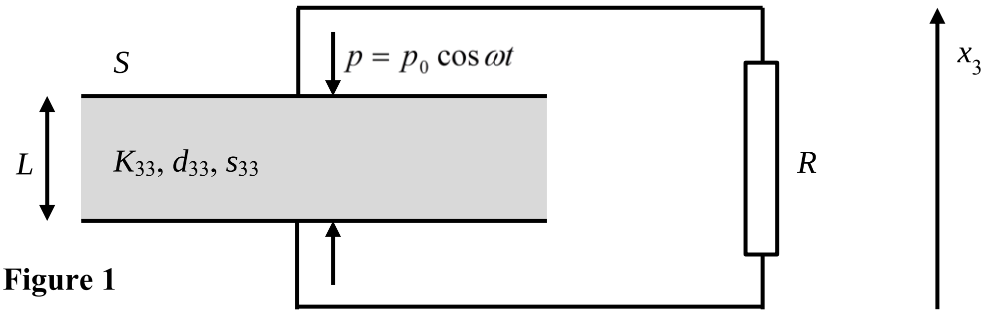

Figure 1

It is convenient to introduce complex variables $\tilde{p}$, $\widetilde{\Delta L}$, etc., such that: $p = \mathrm{Re}[\tilde{p}]$, $\Delta L = \mathrm{Re}[\widetilde{\Delta L}]$, etc. In introduced complex variables, the Young modulus is evaluated as:

$$Y(\omega) = \left|\frac{L\tilde{p}}{\widetilde{\Delta L}}\right|.$$

(i) Find the frequency dependence of $\frac{L\tilde{p}}{\widetilde{\Delta L}}$ in complex designations.

(ii) Using the obtained frequency dependence of $\frac{L\tilde{p}}{\widetilde{\Delta L}}$, show that at frequencies $\omega \ll \omega_0$ ($\omega_0 = 1/RC$) the Young modulus $Y = \left|\frac{L\tilde{p}}{\widetilde{\Delta L}}\right|$ is close to its shot-circuit value, while at $\omega \gg \omega_0$ it is close to its open-circuit value.

All measurements are conducted at constant temperature.

Hints:

(a) use the complex presentations for E_3, D_3, σ_3 ε_3

(b) use the complex presentations for the constitutive electro-mechanic equations

(c) use the complex presentation for the Ohm's law